



1.6.8.2 Additive inverses of complex numbers

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Introduction

- In this topic, we will
 - Define the additive inverse of a complex number
 - Determine how to calculate this inverse





The additive inverse

- Given a complex number $z = \alpha + \beta j$, the additive inverse $-z$ is that number such that

$$z + (-z) = 0$$

- Thus, assume $-z = \gamma + \delta j$, so we have that

$$(\alpha + \beta j) + (\gamma + \delta j) = 0 + 0j$$

$$(\alpha + \gamma) + (\beta + \delta)j = 0 + 0j$$

- Now, two complex numbers are equal if and only if both the real and imaginary components are equal, so thus

$$\alpha + \gamma = 0 \text{ and } \beta + \delta = 0$$

- Thus, $\gamma = -\alpha$ and $\delta = -\beta$, so

$$-z = -\alpha - \beta j$$





The additive inverse

- For example, if $z = 0.7 - 8.4j$, then $-z = -0.7 + 8.4j$, it follows that

$$\begin{aligned} z + (-z) &= (0.7 - 8.4j) + (-0.7 + 8.4j) \\ &= (0.7 - 0.7) + (-8.4 + 8.4)j \\ &= 0 + 0j = 0 \end{aligned}$$

- If $w = -9.6 + 0.1j$, then $-w = 9.6 - 0.1j$, and again we see

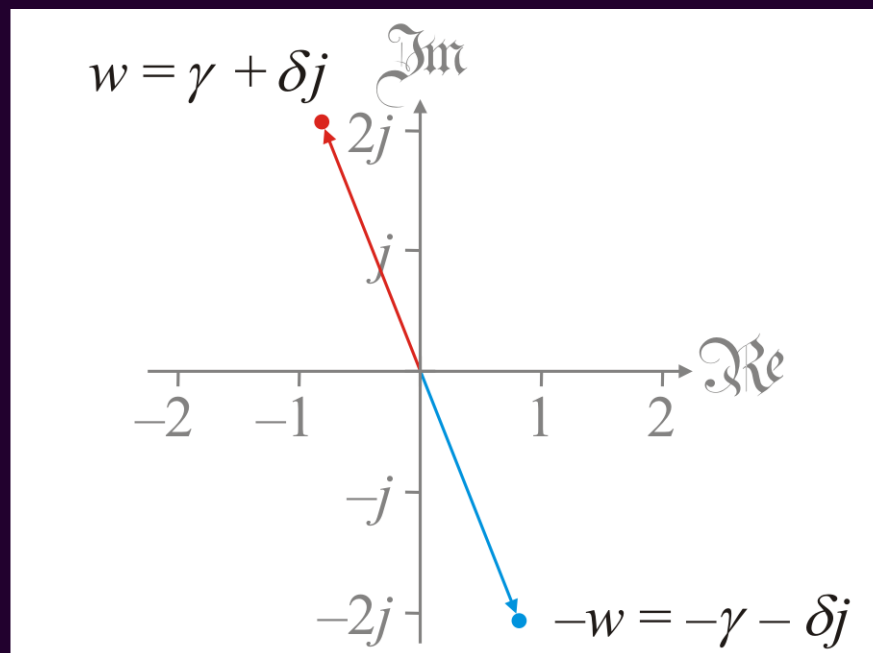
$$\begin{aligned} w + (-w) &= (-9.6 + 0.1j) + (9.6 - 0.1j) \\ &= (-9.6 + 9.6) + (0.1 - 0.1)j \\ &= 0 + 0j = 0 \end{aligned}$$





Complex addition

- Going back to our geometric interpretation:
 - The additive inverse is the reflection of a complex number through 0





Observation

Theorem: $-z = (-1) z$

- What this says is: Given z , we can find $-z$ by multiplying z by -1

Proof: If $z = \alpha + \beta j$,

$$\begin{aligned}\text{it follows that } -z &= -\alpha - \beta j \\ &= (-1)\alpha + (-1)\beta j \\ &= (-1)(\alpha + \beta j) = (-1)z \quad \text{QED}\end{aligned}$$

- Note, we only used the properties of fields
 - Thus, in any field, we can find $-z$ by multiplying z by the additive inverse of 1





Summary

- In this topic, we've introduced the additive inverse
 - Given z , we found $-z$ so that $z + (-z) = 0$
 - We saw the geometric interpretation as a reflection through 0
 - We can find $-z$ by multiplying z by the additive inverse of 1, or -1
 - We will, of course, refer to $-z$ as “minus zed”





References

- [1] https://en.wikipedia.org/wiki/Additive_inverse
- [2] https://en.wikipedia.org/wiki/Complex_number#Field_structure





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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